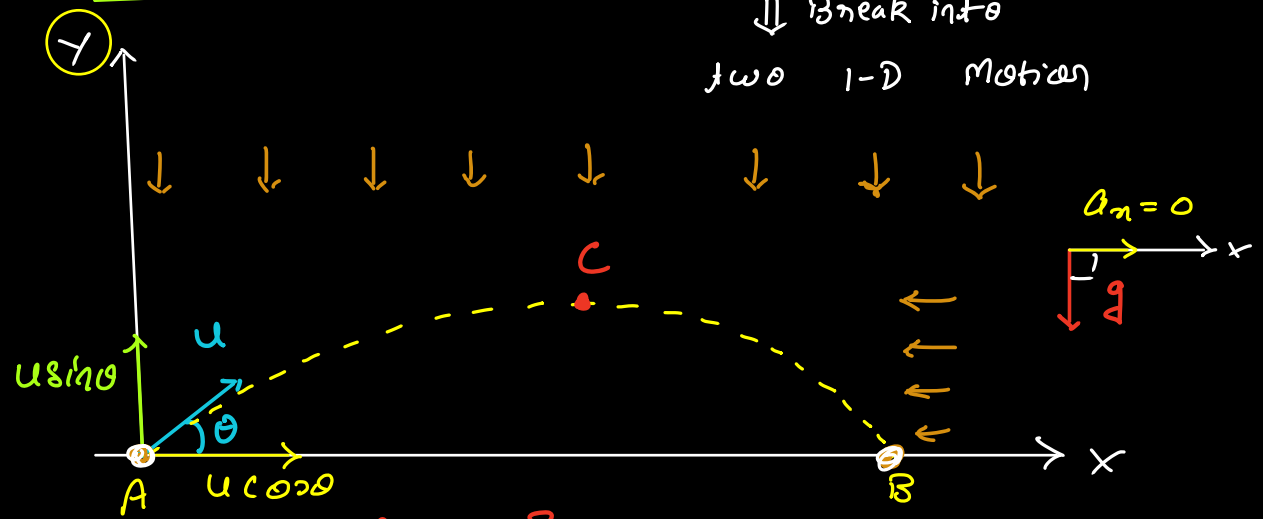
Projectile Motion : (2-D Motion)

⇓ Break into two 1-D Motion



A to B

X-axis	Y-axis
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
$S_x = AB$ (Range (R))	$S_y = 0$
$t = T$	$t = T$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = T\left(u\sin\theta - \frac{gT}{2}\right)$$

$$\Rightarrow u\sin\theta = \frac{gT}{2}$$

$$\Rightarrow \boxed{T = \frac{2u\sin\theta}{g}} \star$$

$$S_x = R = u_x t + \frac{1}{2} a_x t^2$$

$$R = (u\cos\theta)\frac{2u\sin\theta}{g}$$

$$\boxed{T = \frac{2u_y}{g}}$$

$$\boxed{R = \frac{2(u_x)(u_y)}{g}}$$

$$\sin 2\theta = 2\sin\theta\cdot\cos\theta$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

$$\Rightarrow \boxed{R = \frac{2u^2\sin\theta\cdot\cos\theta}{g}}$$

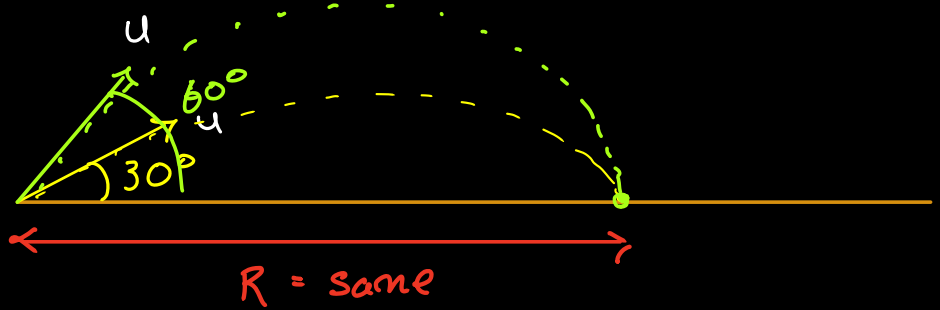
Replace θ by $90-\theta$

θ and $90-\theta$ are complimentary angles

$$\Rightarrow R = \frac{2u^2 \sin(90-\theta)\cdot\cos(90-\theta)}{g}$$

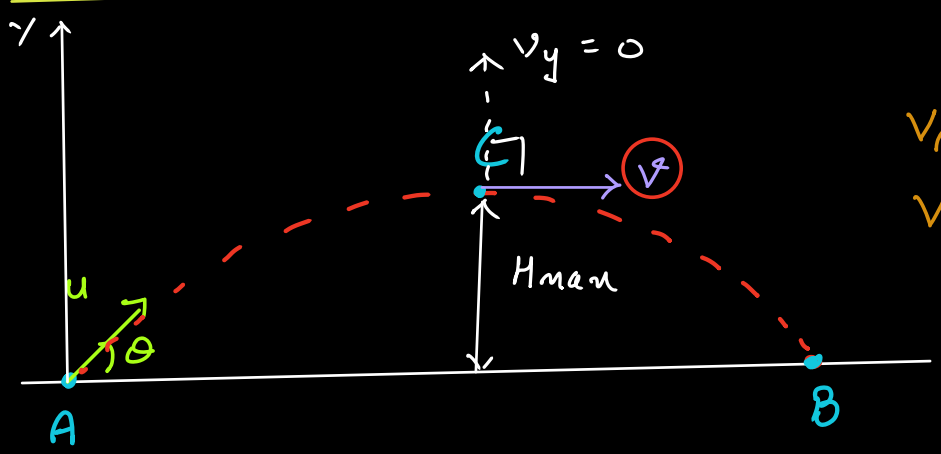
$$\boxed{R = \frac{2u^2 \cos\theta \cdot \sin\theta}{g}}$$

* Keeping speed (u) as same, R = same for complimentary angles !!



$$R = \frac{u^2 \sin 2\theta}{g} \xrightarrow[2\theta = 90^\circ \Rightarrow \theta = 45^\circ]{\text{max } R} R_{max} = \frac{u^2}{g}$$

Maximum Height:



$V_{net} = V \rightarrow$

$V_y = 0$

At highest point ⇒ Y- component of velocity = zero

Y- Direction : A to C

$u_y = u\sin\theta$

$a_y = -g$

$V_y = 0$

$S_y = H_{max}$

t_a = ?? (time of ascent)

$(V_y)^2 = (u_y)^2 + 2(a_y)(S_y)$

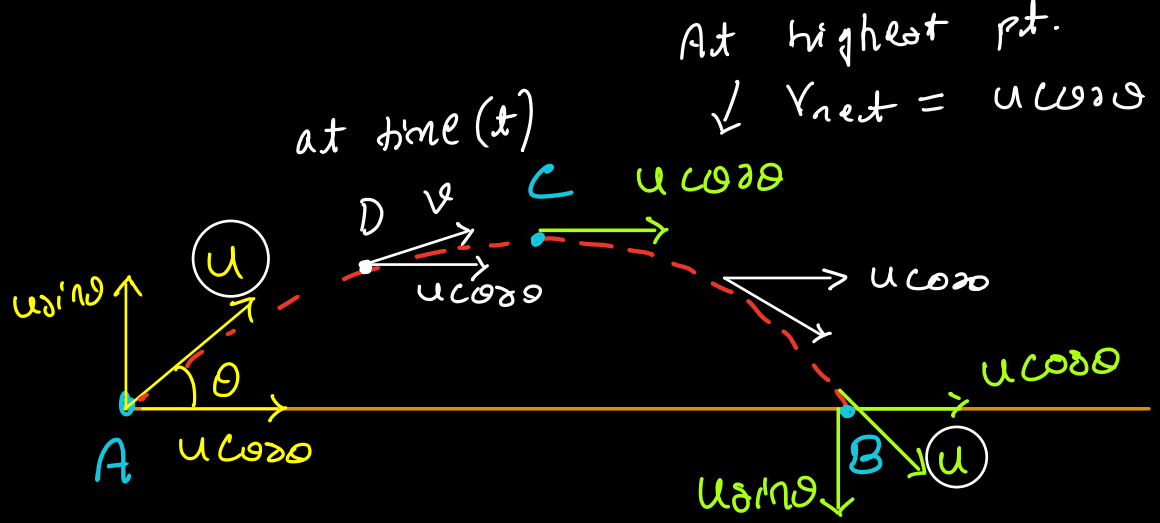
$0^2 = u^2\sin^2\theta - 2g H_{max}$

$$\Rightarrow \boxed{H_{max} = \frac{u^2\sin^2\theta}{2g}} = \frac{(u_y)^2}{2g}$$

$V_y = u_y + a_y t \Rightarrow 0 = u\sin\theta - g t_a \Rightarrow t_a = \frac{u\sin\theta}{g}$

t_d (time of descent) $= \frac{u\sin\theta}{g}$

Velocity ($\vec{v}$) as a function of time:



X	Y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
t	t
$v_x = ?$	$v_y = ?$

$v_x = u_x + a_x t$ (0)

$$v_x = u\cos\theta$$

$v_y = u_y + a_y t$

$$v_y = u\sin\theta - gt$$

$$\vec{v} = (u\cos\theta)\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

* Horizontal component of velocity = Constant = $u\cos\theta$

At pt. B: $t = \frac{2u\sin\theta}{g}$

$\Rightarrow \vec{v}_B = (u\cos\theta)\hat{i} - (u\sin\theta)\hat{j}$

$\Rightarrow \vec{v}_B = (u\cos\theta\,\hat{i}) + u\sin\theta - g\left(\frac{2u\sin\theta}{g}\right)\hat{j}$

At pt. C $\Rightarrow \vec{v}_C = (u\cos 2\theta)\,\hat{i}$

Linear Momentum ($\vec{p}$):

$\vec{p} = m\vec{v}$ Direction is same as that of $\vec{v}$

$$\Delta(\vec{p}) = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$$

A to C $\Rightarrow \Delta\vec{p} = m(\vec{v}_C - \vec{v}_A)$

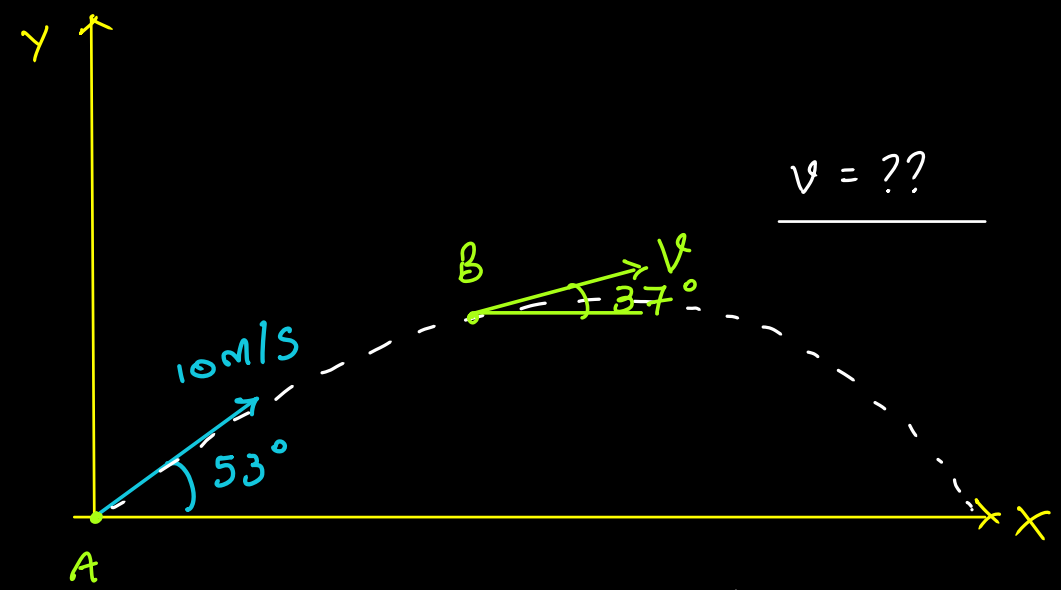
$= m(u\cos 2\theta\,\hat{i} - (u\cos 2\theta\,\hat{i} + u\sin\theta\,\hat{j}))$

$\Delta\vec{p} = (-mu\sin\theta)\,\hat{j}$

A to B $\Rightarrow \Delta\vec{p} = m(\vec{v}_B - \vec{v}_A)$

$= (-2mu\sin\theta)\,\hat{j}$

Example:



Horizontal component of velocity remains same

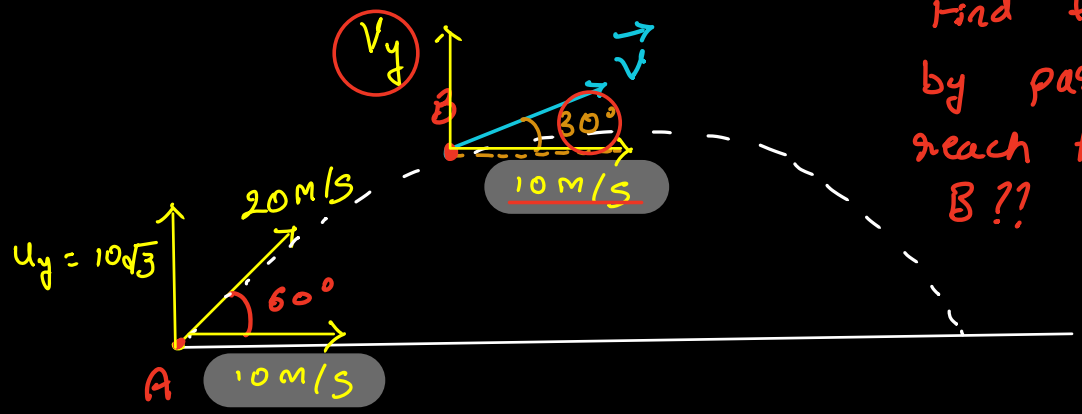
At A: $10\cos 53^\circ = u_x$ $\quad 10\cos 53^\circ = V\cos 37^\circ$

$\Updownarrow$

At B: $V\cos 37^\circ$ $\quad \Rightarrow 10 \times \frac{3}{\cancel{5}} = V \times \frac{4}{\cancel{5}}$

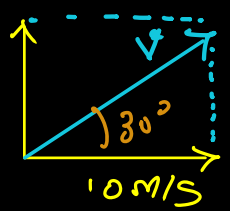
$V = 7.5\ m/s$

Example:



Find time taken by particle to reach from A to B ??

At pt. B



$$\tan 30^\circ = \frac{V_y}{10}$$

$$V_y = \frac{10}{\sqrt{3}} \text{ m/s}$$

Y-Direction :

$u_y = u \sin\theta = 20 \sin 60^\circ$

$= 10\sqrt{3}$ m/s

$a_y = -10$ m/s^2

$V_y = \frac{10}{\sqrt{3}}$ m/s

$t = ??$

$$V_y = u_y + a_y t$$

$$\frac{10}{\sqrt{3}} = 10\sqrt{3} - 10t$$

$$t = \sqrt{3} - \frac{1}{\sqrt{3}} \Rightarrow t = \frac{2}{\sqrt{3}} \text{ secs}$$

will not get anything extra with X-Dir.

X-Dir. :

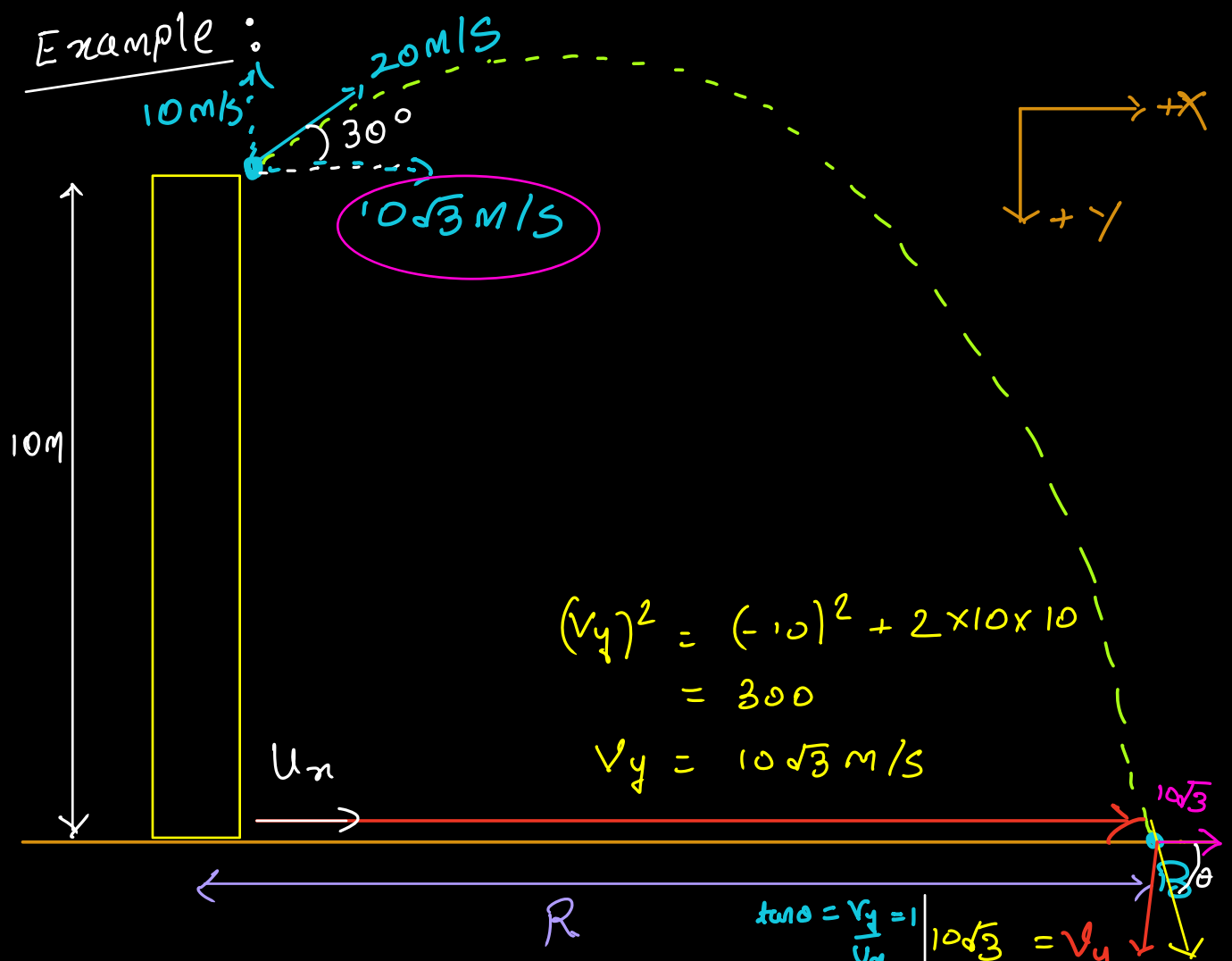
$u_x = 10$ m/s $t = ??$

$V_x = 10$ m/s

$a_x = 0$

$V_x = u_x + a_x t$

Example :



$(v_y)^2 = (-10)^2 + 2 \times 10 \times 10$

$= 300$

$v_y = 10\sqrt{3}$ m/s

$\tan\theta = \frac{v_y}{u_x} = \frac{10\sqrt{3}}{10\sqrt{3}} = 1$

$\theta = 45°$

x	y
$u_x = 10\sqrt{3}$	$u_y = -10$ m/s
$a_x = 0$	$a_y = 10$ m/s^2
$s_x = R = ??$	$s_y = 10$ m
$t = ??$	$t = ??$ $v_y = ??$

$s_y = u_y t + \frac{1}{2} a_y t^2$

$10 = -10t + 5t^2$

$t^2 - 2t - 2 = 0$

$t = \frac{2 \pm 2\sqrt{3}}{2}$

$t = \sqrt{3} + 1$ secs

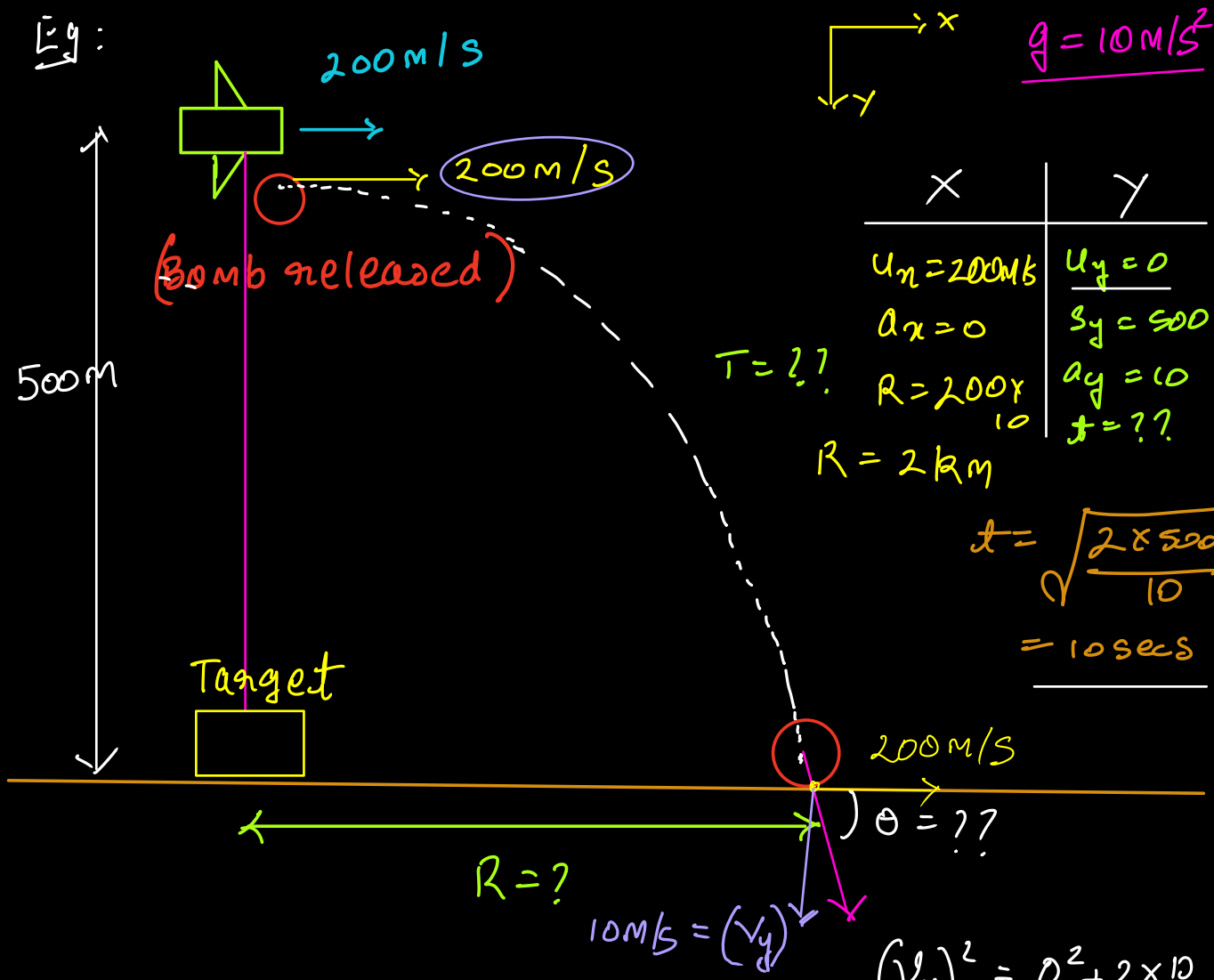
~~$1 - \sqrt{3} = 1 - 1.7 = -ve$~~

$S_x = R = v_x t$

$S_x = 10\sqrt{3}(\sqrt{3}+1) = 30 + 10\sqrt{3}$

$S_x = 47\ m$

Eg:



$g = 10\ m/s^2$

X	Y
$u_x = 200 m/s$	$u_y = 0$
$a_x = 0$	$s_y = 500$
$R = 200 \times 10$	$a_y = 10$
	$t = ??$

$R = 2\ km$

$t = \sqrt{\frac{2 \times 500}{10}}$

$= 10\ secs$

$(V_y)^2 = 0^2 + 2 \times 10 \times 500$

$V_y = 0 + 10 \times 10$

$= 100\ m/s$

$V_y = 100\ m/s$

$\tan\theta = V_y / V_x = 1/2 \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$